B.Tech.

First Semester Examination

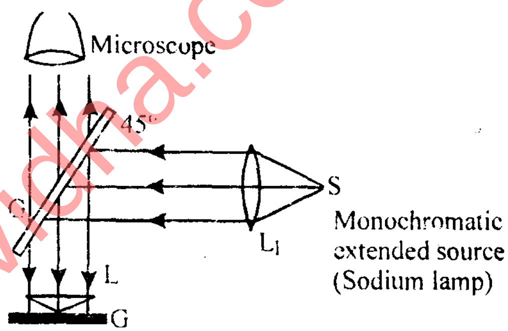
Physics-1 (PHY-101F)

Note : Attempt ***FIVE*** questions in all taking least two questions from each Part. All questions carry equal marks

Part-A

Q. 1. (a) What are Newton's rings? Describe and explain the formation of Newton's rings in reflected mono-chromatic light. Prove that diameter of bright rings are proportional to the square root of add **natural numbers.**

Ans. L is a plano-convex lens of radius of large curvature. The lens with its convex surface is placed on a plane glass plate P. The lens makes contact with the plate at C. Light from the extended monochromatic source such as sodium lamp falls on a glass plate held at an angle of 45° with the vertical. The glass plate G reflects normally a part of incident light towards the air film enclosed by lens L and the glass plate P. A part of the incident light is reflected by the curved surface of lens L and part is transmitted which is reflected back from the plane surface of he plate. These two reflected rays interfere and give rise to an interference pattern in form of circular rings. These rings are localised in an air film and can be seen with a microscope focussed on the film.



Experimental Arrangement for Newton Rings

The path difference between the rays reflected from the top and bottom surface of film is $2\mu t \cos r + \frac{\lambda}{2}$. For air film, $\mu = 1$

For normal incidence $\quad r = 0$

In this case the path difference $\quad = 2t + \frac{\lambda}{2}$

At the point of contact, $\quad t = 0$

Path difference $= \frac{\lambda}{2}$, which is the condition of minimum intensity. Thus, the central spot is dark.

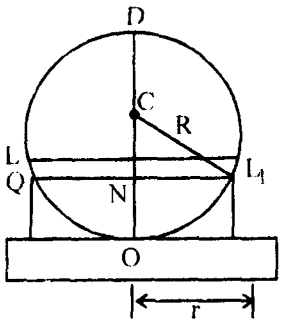
For n^{th} maximum,

$$2t + \frac{\lambda}{2} = n\lambda$$

In this case locus of all the points which have constant thickness is circle.

Thus, fringes are in form of circle.

When we calculate the diameters of bright and dark rings in reflected system, the procedure is as follows. Let LOL' be the lens placed on the glass plate AB. The curved surface LOL' is the part of spherical surface with centre at C. Let R be the radius of curvature and r is the radius of Newton ring corresponding to the constant film thickness t.



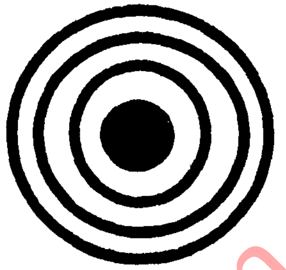


Fig. Newton Rings in Reflected Light

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = \frac{(2n-1)\lambda}{2} \quad \text{for a bright ring } n = 1, 2, 3, ...$$

&

$$2t = n\lambda \quad \text{for dark ring } n = 0, 1, 2, 3, ...$$

From the property of the circle,

$$NP \times NQ = NO \times ND$$

Substituting values,

$$r \times r = t \times (2R - t) = 2Rt - t^2 \simeq 2Rt \text{ approximately.}$$

$$\therefore \quad t = \frac{r^2}{2R}$$

Thus, for bright ring,

$$2.\frac{r^2}{2R} = \frac{(2n-1)\lambda}{2}$$

$$r = \frac{D}{2} \quad \text{where D is diameter}$$

$$\frac{D^2}{4} = \frac{(2n-1)\lambda R}{2}$$

$$D_n = \sqrt{2.(2n-1)\lambda R}$$

$$D_n \, \alpha \sqrt{(2n-1)}$$

i.e., diameter of n^{th} bright ring is proportional to square root of odd natural number.

Q. 1. (b) What change would you expect in interference pattern of Newton's rings if transparent plate below the lens is replaced by a plane mirror?

Ans. If we use a mirror in place of glass plate the rays transmitted by **the** air film and striking **the** main will get reflected and due to superposition of the reflects and transmitted light uniform illumination will result since the conditions for maxima and minima are opposite to each other in the reflected and transmitted beams.

Q. 2. (a) **Differentiate between Fraunhofer and Fresnei diffraction.**

Ans. Difference between Fraunhofer and Fresnei diffraction :

(i) In Fraunhofer diffraction the source of light and the screen are effectively at infinite distances from the diffracting element. In Fresnei diffraction either the source of light or the screen or both are at finite distance from the diffracting element.

(ii) In Fraunhofer diffraction, the incident wavefront is plane while in Fresnei diffraction, the incident wavefront is either spherical or cylindrical.

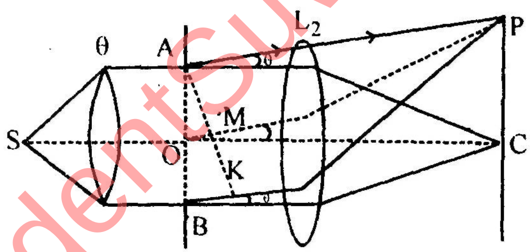
(iii) In Fraunhofer diffraction the observed diffraction pattern is an image of the source modified by diffraction at the diffracting element. In Fresnel diffraction the observed pattern is a projection of diffracting element modified by diffracting effects and the geometry of the source.

(iv) In Fraunhofer diffraction, the effects of a number of diffracting elements can be combined suitably, while it is not so in Fresnei class of diffraction.

Q. 2. (b) Discuss **the phenomenon of Fraunhofer diffraction at a single slit and show that relative** intensities **of** successive maxima are nearly :

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \ldots$$

Ans. Fraunhofer Diffraction at Single Slit:



S is slit source illuminated by monochromatic light of wavelength ####. The light emerges from the lens L_1 as a parallel beam and the wavefront falling on the slit AB of width 'a' is plane. The diffracted light is focussed by another convex lens L_2 on the screen XY.

Let us find resultant intensity at P. Let AK be perpendicular to BK. The path difference between the rays originating from extreme points A and B is given by

$$p = BK = AB\sin\theta = a\sin\theta$$

The phase difference between the extreme rays

$$= \frac{2\pi}{\lambda} \times p = \frac{2\pi}{\lambda} a\sin\theta$$

The phase difference between the waves from two consecutive parts is $\frac{1}{n}$.

$$\frac{2\pi}{\lambda} a\sin\theta = \delta$$

Resultant amplitude at P

$$R = \frac{a'\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} = \frac{a'\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a\sin\theta}{n\lambda}\right)}$$

Let $$\alpha = \frac{\pi a\sin\theta}{\lambda}$$

$$R = \frac{a'\sin\alpha}{\sin\alpha/n} = \frac{a'\sin\alpha}{\alpha/n}$$

$$R = na'\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$$

Resultant intensity at P is given by

$$R^2 = A^2\frac{\sin^2\alpha}{\alpha^2}$$

Maximum and minimum value of I can be found by putting $\frac{dI}{d\alpha} = 0$

Differentiatng above equat.

$$\frac{dI}{d\alpha} = A^2\frac{2\sin\alpha}{\alpha}\cdot\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

This gives either,

$$\frac{\sin\alpha}{\alpha}\cdot\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\frac{\sin\alpha}{\alpha} = 0 \text{ or } \alpha\cos\alpha - \sin\alpha = 0$$

$$\frac{\sin\alpha}{\alpha} = 0 \text{ or } \alpha = \tan\alpha$$

Direction of Minimum Intensity : When $\frac{\sin\alpha}{\alpha} = 0$, Intensity is zero

$$\frac{\sin\alpha}{\alpha} = 0$$

$$\sin\alpha = 0$$

$$\alpha = \pm m\pi$$

$$\frac{\pi a\sin\theta}{2} = \pm m\pi$$

$$a\sin\theta = \pm m\lambda$$

$$m = 1, 2, 3\ldots\ldots \quad m \neq 0.$$

Direction of Maximum Intensity : For this we have,

$$\alpha = \tan\alpha$$

For this solve graphically by plotting curves

$$y = \alpha \text{ and } y = \tan\alpha$$

The first value of a is zero while the remaining values are approximately

$$\frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \quad \alpha = 0 \text{ corresponds to central maxima}$$

The intensity of central principle maximumis

$$I = R^2 = A^2 \frac{\sin^2\alpha}{\alpha^2} = A^2 = I_0$$

Direction of secondary maxima are app. given by

$$\alpha = \pm\frac{(2m+1)\lambda}{2} \qquad m = 1, 2, 3$$

$$a\sin\theta = \pm\frac{(2m+1)\lambda}{2}$$

m= 1,2, 3...... we get position of first, second, third secondary maxima respectively.

The intensity of first secondary maxima

$$I_1 = \frac{A^2\left(\sin\frac{3\pi}{2}\right)^2}{\left(\frac{3\pi}{2}\right)^2} = A^2 . \frac{4}{9\pi^2}$$

The intensity of second secondary maxima

$$I_2 = \frac{A^2 \sin\left(\frac{5\pi}{2}\right)^2}{\left(\frac{5\pi}{2}\right)^2} = A^2 - \frac{4}{25\pi^2}$$

$$I_3 = A^2 - \frac{4}{49\pi^2} = \frac{4}{49\pi^2} I_0$$

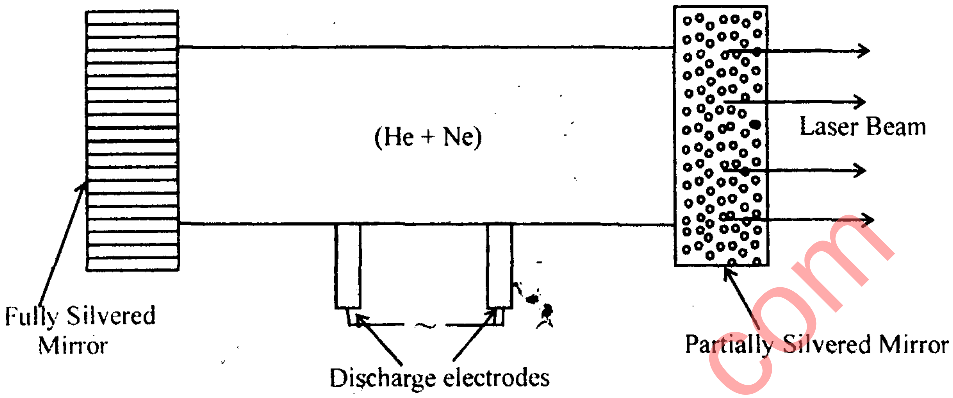
Thus, the relative intensities of successive maxima are,

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} :$$

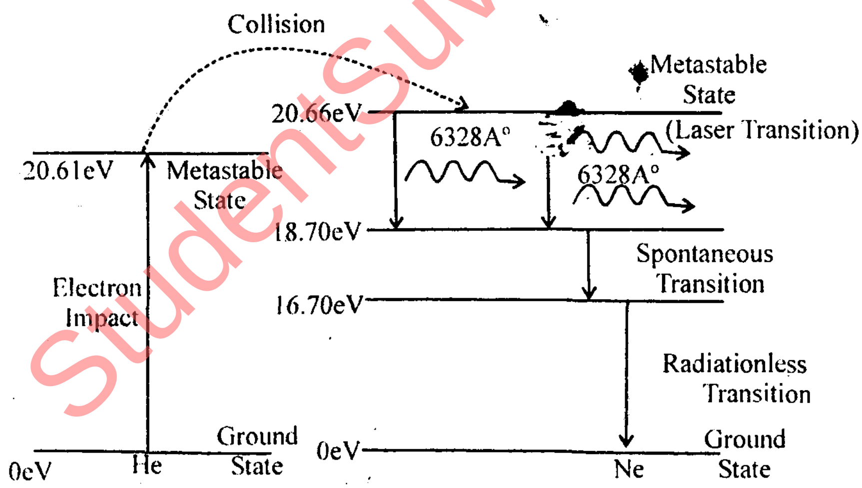
Q. 3. (a) Describe the principle, construction and working of He-Ne gas laser.

Ans. Helium-Neon Laser : The He-Ne laser was the first laser to employ **a** gaseous medium. This is the first laser of any kind in which continuous laser action was demonstrated.

A mixture of about a parts of helium and/part of neon (He : **Ne** = 9 :1) is contained in a glass tube **at a** pressure of about 1 mm of mercury. The ends of the tube are fixed with two **optically plane and parallel mirrors** one partially silvered and the other perfectly silvered. The spacing of the mirrors is equal to an integral number of half wavelengths of the laser light. An electric discharge is produced in the gas mixture by electrode connected to a high frequency electric source.



The electron from the discharge pumps the helium atom from the ground state to a metastable state at an energy level of 20.61 eV. The excited helium atom now collides inelastically with a neon atom and transfers its energy to it. Incidently the neon atom has got an excited state at the energy level 20.66eV the 0.05eV of additional energy being provided by the kinetic energy of atom after colosion. Thus, the neon atom is raised to this level and the helium atom, after transferring its energy to then neon atom goes back to its ground state. Thus, He atom help in achieving population inversion for the Ne atoms.



Below the metastable state, the neon atom has another short lived excited state at the energy level of 18.70eV. When an excited Ne atom passes spontaneously from the metastable state at 20.66eV to an excited state at 18.70eV, it emits a 6328A° photon. This photon travels through the gas mixture, and if it is moving parallel to the axis of tube, is reflected back and forth by the mirror ends until it stimulates an excited Ne atom and cause it to emit a fresh 6328A° photon in phase with the stimulating photon. This stimulating transition from 20.66eV levels to 18.70eV level is the laser transition. This process is continued and a beam of coherent radiations builds up in the tube. When this beam becomes sufficiently intense, a portion of it escapes out through the partially silvered end.

From the 18.70eV level the Ne atom passes down spontaneously to a lower metastable state of energy

16.70eV emitting incoherent light and finally to the ground state through collision with the tube walls. Thus, the tlnal transition is radiationless.

Q. 3. (b) Discuss Einstein's co-efficients. Derive **relation between them.**

Ans. Consider an assembly of atoms in thermal equilibrium at temperature. T with radiation of frequency v and eneru\ density u(v). Let N_1 and N_2 be number of atoms in energy states 1 and 2 respectively, at any instant. The number of atoms in state 1 that can absorb a photon and give rise to absorption per unit time

$$= N_1 P_{12} = N_1 B_{12} u(\nu)$$

Similarly $$N_2 P_{12} = N_2 (A_{21} + B_{21} u(\nu))$$

For equilibrium,

$$N_1 P_{12} = N_2 P_{21}$$

$$N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$$

$$= N_2 A_{21} + N_2 B_{21} u(\nu)$$

$$u(\nu)[N_1 B_{12} - N_2 B_{21}] = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$= \frac{A_{21}}{B_{21}} \frac{1}{\frac{N_1}{N_2}\left(\frac{B_{12}}{B_{21}}\right) - 1}$$

According to Boltzmann distribution law.

$$N_1 = N_0 e^{-E_1/kT} \text{ and } N_2 = N_0 e^{-E_2/kT}$$

$$\frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-(E_2 - E_1)/kT}$$

But $$E_2 - E_1 = h\nu$$

$$\frac{N_2}{N_1} = e^{-h\nu/kT}$$

$$\frac{N_1}{N_2} = e^{h\nu/kT}$$

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{e^{h\nu/kT} \frac{B_{12}}{B_{21}} - 1}$$

Comparing with

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} e^{\frac{1}{h\nu/kT}} - 1$$

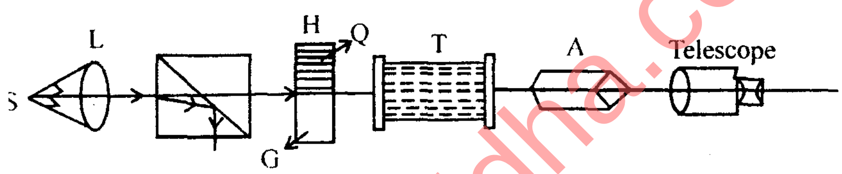
$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \text{ and } \frac{B_{12}}{B_{21}} = 1$$

$$B_{21} = B_{12}$$

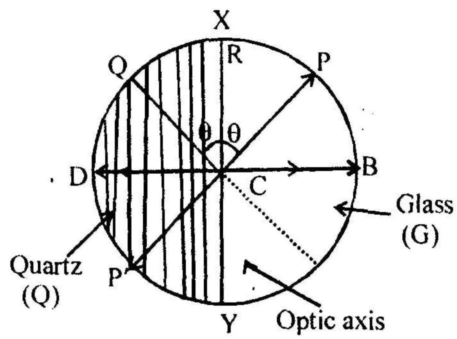
Q. 4. (a) Describe the construction and working of Laurent's half shade polarimeter. Explain fully the action of half shade device.

Ans. Half Shade Polarimeters :

Apparatus : The experimental arrangement is shown in fig. S is a source of monochromatic light placed at focus of a convex lamp L. The beam, rendered parallel by lens L, falls on a Nicol prism (called polariser) P. After passing through polariser P the light becomes plane polarised. This polarised light beam passes through a half shade device H & then through a tube T containing optically active solution. The transmitted light passes through another Nicol A which can be rotated about the direction of propagation of light. The light emerging from the analyser is observed through a telescope.



Action of Half-Shade Plate : Half shade plate H is a combination of two semi-circular plates XBY & XDY. The plate XDY is of quartz & is cut parallel to optic axis while the plate XBY is of glass. Both the plates are joined along the diameter XY. The thickness of quartz plate is such that it introduces a path difference #### or a phase difference ### between ordinary & extraordinary vibrations. The thickness of glass plate is such that it absorbs same amount of light as the quartz plate.



The light emerging from the polarised (P) is plane polarized & falls normally on the half-shade plate. One half of this passes through Q & other through G. The light falling on quartz plate is broken up into two components parallel & perpendicular to optic axis. The two components travel along the same rection but with different speeds. Thus, the plane of vibration of light emerging from quartz is inclined **to** the plane **of** vibration of light emerging from glass at an angle 2###. Thus, on emergence from the combination, we have two plane polarized beams, one passing out of the glass plate & other from quartz plate.

Q. 4. (b) Calculate the numerical aperture and acceptance angle of an optical fibre. Given refrac tive index of fibre core = 1.5 and refractive index of cladding = 1.47.

Ans.
$$N_f = 1.5$$

$$N_c = 1.47$$

$$N.A = \sqrt{N_f^2 - N_c^2}$$

$$= \sqrt{(1.5)^2 - (1.47)^2}$$

$$= 0.2441$$

Acceptance angle $\sin\theta = N.A$

$$\theta = \sin^{-1}(0.2441)$$

$$= 14°8'$$

Part-B

Q. 5. Establish the differential equation for forced harmonic oscillator and discuss the condition for resonant amplitude. Show dependence of the amplitude as a function of driving frequency.

Ans. Forced Harmonic Oscillator: Consider a particle of mass **m oscillator about equilibrium position.** The spring constant is k and damping constant is k. The particle is **driven by periodic force F = F_0 sin pt**

Force acting on the particle are,

(i) Restoring free due to spring

$$= -k_x$$

(ii) Damping force $= \dfrac{-bdx}{dt}$

(iii) Applied periodic force $= F_0 \sin pt$

: Net force on particle,

$$F = F_0 \sin pt - \frac{bdx}{dt} - k_x$$

Equation of harmonic solution,

$$F = m_a = \frac{md^2x}{dt^2} = F_0 \sin pt - \frac{bdx}{dt} - k_x$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin pt$$

$$\frac{b}{m} = 2r \frac{k}{m} = \omega_0^2 \text{ and } \frac{F_0}{m} = f_0$$

$$\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega_0^2 x = f_0 \sin pt \qquad \text{...(1)}$$

Complete solution of this equation consist of two parts.

(i) **A Complementary Function** : When RHS is zero.

$$\frac{d^2x}{dt^2} + 2r\frac{dx}{dt} + \omega_0^2 x = 0$$

This gives transient part which dies away with time.

(ii) The Steady State Function : Let solution of above equation be

$$x = A\sin(pt - \phi)$$

$$\frac{dx}{dt} = pA\cos(pt - \phi)$$

$$\frac{d^2x}{dt^2} = -p^2 A\sin(pt - \phi)$$

Putting these values in above equation

$$A\left(\omega_o^2 - p^2\right)\sin(pt - \phi) + 2rpA\cos(pt - \phi) = f_0\sin(pt - \phi)\cos\phi + f_0\cos(pt - \phi)\sin\phi$$

Equating

$$A\left(\omega_o^2 - p^2\right) = f_0\cos\phi \qquad \text{...(2)}$$

$$2rpA = f_0\sin\phi \qquad \text{...(3)}$$

Squaring and adding equations (2) and (3)

$$A^2\left[\left(\omega_o^2 - p^2\right)^2 + 4r^2p^2\right] = f_0^2$$

$$A = \frac{f_0}{\sqrt{\left(\omega_o^2 - p^2\right)^2 + 4r^2p^2}} \qquad \text{...(5)}$$

$$\tan\phi = \frac{2rp}{\omega_o^2 - p^2}$$

Putting value of A in equation (3)

$$x = \frac{f_0}{\sqrt{\left(\omega_0^2 - p^2\right)^2 + 4r^2p^2}}\sin(pt - \phi) \qquad \text{...(6)}$$

Amplitude Resonance : Equation (5) shows that the amplitude of forced oscillations depends upon $\left(\omega_0^2 - p^2\right)$ which is then depends upon the difference between the driving frequency p and natural frequency ω_0 of the oscillator. Smaller this difference, larger the amplitude.

Amplitude will be maximum when the denominator in equation (5) is maximum. This will be so when.

$$\frac{d}{dp}\left[\left(\omega_o^2 - p^2\right)^2 + 4r^2p^2\right] = 0$$

$$= \left(\omega_o^2 - p^2\right)(-2p) + 4r^2(2p) = 0$$

$$\omega_0^2 - p^2 = 2r^2$$

$$p = \sqrt{\omega_0^2 - 2r^2}$$

Write p_r, f_n p at resonance we have

$$p_r = \sqrt{\omega_0^2 - 2r^2}$$

Amplitude of forced oscillator is maximum when frequency $p/2\pi$ of the impressed force is,

$$\frac{p_r}{2\pi} = \frac{\sqrt{\omega_0^2 - 2r^2}}{2\pi}$$

Which is slightly less than natural frequency $\omega_0/2\pi$ as well as than damped frequency $\frac{\sqrt{\omega_0^2 - r^2}}{2\pi}$ of the system.

Q. 6. (a) Derive Maxwell's equations and give their physical interpretation.

Ans. The four fundamental equations of electrons of electro magnetism known as Maxwell's equation can be written in differential form as.

(i) $\vec{\nabla}.\vec{D} = \delta$ (differential form of Gauss law in electrostatics).

(ii) $\vec{\nabla}.\vec{B} = 0$ (differential form of Gauss law in magnetostatics which is usually said to represent the fact that isolated magnetic poles do not exist).

(iii) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ (differential form of Faraday's law of electromagnetic inductance).

(iv) $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ (Maxwell's modification of Ampere's law to include time varying field).

In above equations, the different notations have following meanings :

$\vec{D}$ = Electric displacement vector in $coul/m^2$.

δ = Charge density in $coul/m^2$.

$\vec{B}$ = Magnetic inductance in weber/ m^2.

$\vec{E}$ = Electric field intensity in volt/m

$\vec{H}$ = Magnetic field intensity in amp/m-turn

$\vec{J}$ = Current density in amp/m^2.

(i) Derivation of First Equation $div\vec{J} = \vec{\nabla}.\vec{D} = \delta$: Let us consider a surface s bounding a volume 'V' in dielectric medium. In a dielectric medium total charge consists of free charge plus polarization charge. If δ and δ_p are the charge densities of the free charge and polarisation charge at a point in a small volume element d_{v_1} then Gauss law can be expressed as :

$$\int_s \vec{E}\vec{ds} = \frac{1}{\varepsilon_0}\int_v (\delta + \delta_p)dv$$

But polarization charge density is defined as,

$$\delta_p = -\text{div}\vec{P}$$

Therefore above equation takes the form :

$$\int_s \vec{E}.\vec{ds} = \frac{1}{\varepsilon_0}\int_v \left(\delta - \text{div}\vec{P}\right)dv$$

Or
$$\int_s \varepsilon_0 \vec{E}.\vec{ds} = \int_v \delta.dv - \int_v \text{div}\,\vec{P}\,dv$$

Using Gauss divergence the theorem to change surface integral into volume integral, we get

$$\int_v \text{div}\left(\varepsilon_0 \vec{E}\right)dv = \int_v \delta.dv - \int_v \text{div}\,\vec{P}.dv$$

Or
$$\int_v \text{div}\left(\varepsilon_0 \vec{E} + \vec{P}\right)dv = \int_v \delta.dv$$

But $\varepsilon_0\,\vec{E} + \vec{P} = \vec{D}$, $\vec{D}$ being electric displacement vector,

∴
$$\int_v \left(\text{div}\,\vec{D}\right)dv = \int_s \delta.dv$$

Or
$$\int_v \left(\text{div}\,\vec{D} - \delta\right)dv = 0$$

Since this equation is true for all volumes, therefore the integrand must vanish i.e.,

$$\text{div}\vec{D} - \delta = 0$$

Or
$$\text{div}\vec{D} = \delta$$

i.e.,
$$\boxed{\vec{\nabla}.\vec{D} = \delta}$$

(ii) Derivation of Second Equation div $\vec{\mathbf{B}} = \vec{\nabla}.\vec{\mathbf{B}} = \mathbf{0}$: Since isolated magnetic poles and magnetic currents due to them have no physical significance, therefore magnetic lines of force in general are either closed curves or go off to infinity. It means that the flux of magnetic induction B across any closed surface is always zero, i.e.,

$$\oint_s \vec{B}.\vec{ds} = 0$$

Using Gauss div., theorem to change surface integral into volume integral, we get

$$\int_v \text{div}\vec{B}dv = 0$$

As the surface bounding the volume is arbitrary, therefore this equation holds only if the integrand vanishes i.e.,

$$\text{div}\,\vec{B} = 0 \text{ or } \boxed{\vec{\nabla}.\vec{D} = 0}$$

(iii) Derivation **of Third Equation curl** $\vec{\mathbf{E}} = \vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$: According to Faraday's law of electromagnetic induction, we know that e.m.f. induced in a closed loop is defined as negative rate of change of magnetic flux i.e..

$$e = \frac{-\partial \phi}{\partial t}$$

But magnetic flux

$$\phi = \int_s \vec{B}.\vec{ds}$$ where s is any surface having loop as boundary

$$e = \frac{-\partial}{\partial t}\int_s \vec{B}.\vec{ds}$$

$$= -\int_s \frac{\partial \vec{B}}{\partial t}.\vec{ds} \qquad ...(1)$$

Assuming that surface's' is fixed in space and only $\vec{B}$ changes with time.

Now, e.m.f. is defined as the amount of work done in moving a unit charge around the closed loop C. Thus, if $\vec{E}$ is the electric field intensity at a point, the work done in moving a unit charge through a small displacement $\vec{de}$ is, $\vec{E}.\vec{dl}$ so.

$$e = -\int_c \vec{E}.\vec{dl} \qquad \textbf{...(2)}$$

Comparing equations (1) and (2), we get

$$\int_c \vec{E}.\vec{dl} = -\int_s \frac{\partial \vec{B}}{\partial t}.\vec{ds} \qquad \textbf{...(3)}$$

Transforming line integral into surface integral using Stoke's theorem,

$$\int_s \text{curl}.\vec{E}.\vec{ds} = -\int_s \frac{\partial \vec{B}}{\partial t}.\vec{ds}$$

Or $$\int_s \left(\text{curl}\,\vec{E} + \frac{\partial B}{\partial t}\right).\vec{ds} = 0 \qquad \textbf{...(4)}$$

Since surface is arbitrary, therefore equation (4) holds only if the integrand vanishes, i.e..

$$\text{curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

Or $$\boxed{\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}}$$

(iv) Derivation of Fourth Equation C $\vec{H} = \vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ **:** m Ampere's circuital law, we have

$$\int_c \vec{H}.\vec{dl} = I$$

Or $$\int_c \vec{H}.\vec{dl} = \int_s \vec{J}.\vec{ds} \qquad \text{...(1)}$$

Where 's' is the surface bounded by closed path C changing the line integral into surface integral by Stoke's theorem, we get.

$$\int_c \text{curl}\,\vec{H}.\vec{ds} = \int_s \vec{J}.\vec{ds}$$

i.e.. $$\int_c \left(\text{curl}\,\vec{H}.\vec{J}\right)\vec{ds} = 0 \qquad \text{...(2)}$$

As the surface is arbitrary therefore integrand must vanish i.e.,

$$\text{curl}\,\vec{H} - \vec{J} = 0$$

Or $$\text{curl}\,\vec{H} = \vec{J} \qquad \text{...(3)}$$

Let us now examine the validity of this equation for time varying fields. If we take the divergence of both sides of equation (3). Then,

$$\text{div}\left(\text{curl}\,\vec{H}\right) = \text{div}\vec{J}$$

Or $$\text{div}\vec{J} = 0 \qquad \text{...(4)}$$

Because divergence of curl of any vector is zero.

Now equation of continuity is

$$\text{div}\vec{J} + \frac{\partial\delta}{\partial t} = 0$$

i.e.. $$\text{div}\vec{J} = -\frac{\partial\delta}{\partial t} \qquad \text{..(5)}$$

According to this equation $\text{div}\vec{J} = 0$ only if $\frac{\partial\delta}{\partial t} = 0$, i.e., charge density is static. Thus, we conclude **that** Ampere's law as stated in equation (3) is valid only for steady state conditions and is insufficient for the cases of time varying fields. To include time varying fields Maxwell assumed that equation (3) is not complete but should be have something also to it. Let this something be denoted by $\vec{J}_d$, the equation (3) becomes :

$$\text{curl}\,\vec{H} = \vec{J} + \vec{J}_d \qquad \text{...(6)}$$

In order to identify $\vec{J}_d$. Let us take the div of equation (6).

i.e., $$\text{div}\left(\text{curl}\,\vec{H}\right) = \text{div}\left(\vec{J} + \vec{J}_d\right)$$

Or $$0 = \text{div}\left(\vec{J} + \vec{J}_d\right) \qquad \left[\text{as div}\left(\text{curl}\,\vec{H}\right) = 0\right]$$

Or $$\text{div}\,\vec{J}_d = -\text{div}\vec{J} = \frac{\partial\delta}{\partial t}$$

$$\left[\text{since from equation (3) div}J = \frac{-\partial\delta}{\partial t}\right] \qquad \text{...(7)}$$

But Gauss theorem in differential form gives

$$\operatorname{div}\vec{D} = \delta \qquad \text{...(8)}$$

Using equation (7) & equation (8), we get

$$\operatorname{div}\vec{J}_d = \frac{\partial}{\partial t}\left(\operatorname{div}\vec{D}\right)$$

$$= \operatorname{div}\frac{\partial \vec{D}}{\partial t}$$

This gives

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} \qquad \text{...(a)}$$

Therefore the modified form of Ampere's circuital law.

$$\operatorname{curl}\ \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

The term which Maxwell added to Ampere's law viz., $\frac{\partial \vec{D}}{\partial t}$ to include time varying fields is known as displacement current because it arises with electric displacement vector $\vec{D}$ changes with time.

Physical Significance **of Maxwell's Equations** : By means of Gauss's & Stake's theorems we can put the field equations in integral form and hence obtain their physical significance.

(i) **Maxwell's First Equation** is $\vec{\nabla}.\vec{D} = \delta$: Integrating this over an arbitrary volume V, we get

$$\int_v \nabla.D\,dv = \int_v \delta\,dv$$

Changing volume integral in L.H.S. of above equation into surface integral by Gauss div. theorem, we get

$$\int_s D.ds = \int_v \delta\,dv \qquad \text{...(1)}$$

Where s is the surface which bounds volume V. Equation (1) represents Maxwell's first equation $\nabla.D = \delta$ in integral form.

It signifies that : The net outward flux of electric displacement vector through the surface enclosing a volume is equal to the net charge contained within that volume.

(ii) Maxwell's Second **Equation is** $\nabla.\mathbf{B} = \mathbf{0}$ '• Integrating this over an arbitrary volume V, we get

$$\int_v \nabla.B = 0$$

Using Gauss divergence theorem to change volume integral into surface integral, we get

$$\int_s B.ds = 0 \qquad \text{...(2)}$$

Where s is the surface which bounds volume V. Equation (2) represents Maxwell's second equation in integral form & signifies that:

(iii) Maxwell's Third Equation is $\nabla \times E = -\frac{\partial B}{\partial t}$: Integrating above equation over a surface s bounded by a curve C, we get,

$$\int_s (\nabla \times E).ds = -\int_s \frac{\partial B}{\partial t}.ds$$

Using Stoke's theorem to convert surface integral on L.H.S. of above equation into line integral along the boundary of C, we get

$$\int_c E.dl = -\frac{\partial}{\partial t}\int_s B.ds \quad ...(3)$$

Equation (3) represents Maxwell's third equation in integral form and signifies that :

The electromotive force around a closed path is equal to negative rate of change of magnetic flux linked with the path.

***(iv)* Maxwell's Fourth Equation is $\nabla \times H = J + \frac{\partial D}{\partial t}$** : Taking surface integral over surface s bounded by curve C, we obtain

$$\int_s (\nabla \times H).ds = \int_s \left(J + \frac{\partial D}{\partial t}\right).ds$$

Using Stoke's theorem to convert surface integral on L.H.S. of above equation into line integral, we get,

$$\oint_c H.dl = \int_s \left(J + \frac{\partial D}{\partial t}\right).ds \quad ...(4)$$

This equation represents Maxwell's fourth equation in integral form and signifies that :

The magnetomotive force $\left(m.m.f. = \int_c H.dl\right)$ around a closed path is equal to the conduction current plus displacement current through any surface bounded by the path.

Q. 6. (b) What is Poynting vector? Explain.

Ans. Poynting Theorem : Maxwell's equations are :

$$\vec{\nabla}.\vec{D} = \delta \quad ...(1)$$

$$\vec{\nabla}.\vec{B} = 0 \quad ...(2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad ...(3)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad ...(4)$$

Taking scalar product of equation (3) with $\vec{H}$ and equation (4) with $\vec{E}$, we get

$$\vec{H}.(\vec{\nabla}\times\vec{E}) = -\vec{H}\frac{\partial\vec{B}}{\partial t} \qquad ...(5)$$

& $$\vec{E}.(\vec{\nabla}\times\vec{H}) = \vec{E}.\vec{J} + \vec{E}\frac{\partial\vec{D}}{\partial t} \qquad ...(6)$$

Now subtracting equation (6) from (5), we get

$$\vec{H}.(\vec{\nabla}\times\vec{E}) - \vec{E}.(\vec{\nabla}\times\vec{H}) = -\vec{H}.\frac{\partial\vec{B}}{\partial t} - \vec{E}.\frac{\partial\vec{D}}{\partial t} - \vec{E}.\vec{J}$$

$$= -\left(\vec{H}.\frac{\partial\vec{B}}{\partial t} + \vec{E}.\frac{\partial\vec{D}}{\partial t}\right) - \vec{E}.\vec{J} \qquad ...(7)$$

Using vector identity

$$\vec{\nabla}.(\vec{E}\times\vec{H}) = \vec{H}.(\vec{\nabla}\times\vec{E}) - \vec{E}.(\vec{\nabla}\times\vec{H})$$

We get $$\vec{\nabla}.(\vec{E}\times\vec{H}) = -\left(\vec{H}.\frac{\partial\vec{B}}{\partial t} + \vec{E}.\frac{\partial\vec{D}}{\partial t}\right) - \vec{E}.\vec{J} \qquad ...(8)$$

Now, if the medium is linear so that the relation,

$\vec{B} = \mu\vec{H}$ and $\vec{D} = \varepsilon\vec{E}$, then we may write,

$$\vec{E}.\frac{\partial\vec{D}}{\partial t} = \vec{E}.\frac{\partial}{\partial t}(\varepsilon\vec{E}) = \frac{1}{2}\varepsilon\frac{\partial}{\partial t}(E^2) = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{E}.\vec{D}\right)$$

& $$\vec{H}.\frac{\partial\vec{B}}{\partial t} = \vec{H}.\frac{\partial}{\partial t}(\mu\vec{H}) = \frac{1}{2}\mu\frac{\partial}{\partial t}(H^2) = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{H}.\vec{B}\right)$$

Therefore,

$$\vec{\nabla}.(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}(\vec{E}.\vec{D} + \vec{H}.\vec{B})\right] - \vec{J}.\vec{E} \qquad ...(9)$$

Each term in the above equation has certain physical significance which may be seen by integrating equation (9) Over a value V bounded by surface s. Thus,

$$\int_V \vec{\nabla}.(\vec{E}\times\vec{H})dv = -\int_V \left\{\frac{\partial}{\partial t}\frac{1}{2}(\vec{E}.\vec{D} + \vec{H}.\vec{B})\right\}dv - \int_V \vec{J}.\vec{E}\,dv$$

Using Gauss divergence theorem to change volume integral on L.H.S. of above equation into surface integral, we get

$$\oint_s (\vec{E}\times\vec{H}).d\vec{s} = -\frac{d}{dt}\int_V \frac{1}{2}(\vec{E}.\vec{D} + \vec{H}.\vec{B})dv - \int_V \vec{J}.\vec{E}\,dv$$

Rearranging this equation, we get

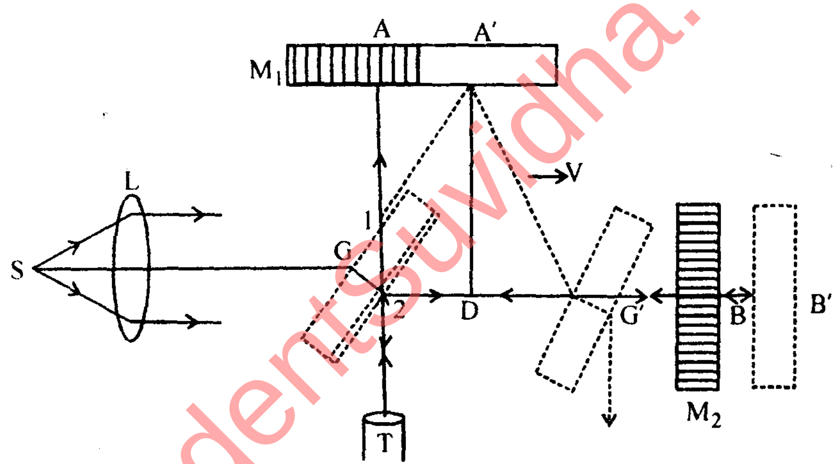
$$-\int_V \vec{J}.\vec{E}\,dv = \frac{d}{dt}\int_V \frac{1}{2}(\vec{E}.\vec{D} + \vec{H}.\vec{B})dv + \oint_s (\vec{E}\times\vec{H}).d\vec{s}$$

This is Poynting theorem which states that the time rate of change of electromagnetic energy **within a** certain volume plus time rate of the energy flowing out through the boundary surface, is equal to the **power** transferred into the electromagnetic field.

This is the statement of conservation of energy in electromagnetism or electromagnetic field.

Q. 7. (a) **Explain Michelson-Morley experiment. How is its negative result of the experiment interpreted?**

Ans. Experimental Arrangement: The experimental arrangement is shown in fig., light from a monochromatic extended source S after being rendered parallel by a collimating lens L, falls on the semisilvered glass plate G inclined at an angle 45° to the beam. It is divided into two parts, one being reflected from the semisilvered surface of G giving rise to ray 1 which travels towards mirror M_1 and the other being transmitting giving rise to ray 2 which travels towards mirror M_2. The two ray fall normally on mirror M_1 and M_2 respectively and are reflected back along their original paths. The reflected rays again meet at the semisilvered surface of glass plate G and enter the telescope where interference pattern is obtained. The optical distances of the mirror M_1 and M_2 from G are made equal with the help of a compensating plate.



If the apparatus is at rest in either, the two reflected rays would take equal time to return the glass plate G. But actually the whole apparatus is moving along with the earth. Let us suppose that the direction of motion of earth is in the direction of the initial beam. Due to the motion of the earth, the optical paths traversed by both the rays are not the same. The reflections at mirrors M_1 and M_2 do not take place at A and B but at A' and B' respectively as shown in fig. Thus, the times taken by the two rays to travel to the mirrors and back to G will be different in this case.

Theory : Let the two mirrors M_1 and M_2 be at an equal distance 1 from the glass plate G. Further let C and V be the velocities of light and apparatus or earth respectively. It is obvious from fig. that the reflected ray 1 from glass plate G strikes the mirror M_1 and A' and not at A due to the motion of the earth. The total path of the ray from G to A' and back to G' will be GA^1G'. But by the law of reflection.

$$GG' = 2GD = 2AA'$$

$\therefore$ $$GA'G' = GA' + A'G' = 2GA' \qquad ...(1)$$

But $$(GA')^2 = (GD)^2 + (A'D)^2 \qquad ...(2)$$

Or $$(GA')^2 = (AA')^2 + (A'D)^2 \quad ...(3)$$

If t be the time taken by the ray to move from G to A', then from equation (3), we have

$$(Ct)^2 = (vt)^2 + (l)^2$$

Or $$t^2(C^2 - v^2)^2 = l^2$$

$$t = \frac{l}{\sqrt{C^2 - v^2}}$$

If t be the time taken by the ray of travel the whole path GA' G', then

$$t_1 = 2t = \frac{2l}{\sqrt{C^2 - v^2}} = \frac{2l}{C\left(1 - \frac{v^2}{C^2}\right)^{1/2}}$$

$$= \frac{2l}{C}\left[1 + \frac{v^2}{2C^2}\right] \quad ...(4)$$

Now consider the case of the transmitted ray 2 which is moving longitudinally towards mirror M_2 • It has a velocity $[C-v]$ relative to the apparatus when it is moving from G to B. During its return journey, its velocity relative to apparatus is $[C+v]$. If t_2 be the total time taken by the longitudinal ray to reach G', then

$$t_2 = \frac{l}{(C-v)} + \frac{l}{(C+v)} \qquad (\because GB = G'B' = l)$$

$$\therefore \quad t_2 = \frac{2lC}{(C^2 - v^2)} = \frac{2lC}{C^2(1 - v^2/C^2)}$$

$$= \frac{2l}{C}\left[1 + \frac{v^2}{C^2}\right] \quad ...(5)$$

Thus, the difference in times of travel of longitudinal and transverse journey is

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{C}\left(1 + \frac{v^2}{C^2}\right) - \frac{2l}{C}\left(1 + \frac{1}{2}\frac{v^2}{C^2}\right)$$

$$= \frac{2l}{C} \cdot \frac{v^2}{2C^2} = \frac{lv^2}{C^3} \quad ...(6)$$

Hence, the path difference between two rays in given by

$$n = \frac{\text{path difference}}{\text{wavelength}} = \frac{C(\Delta t)}{\lambda}$$

$$n = \frac{Clv^2}{C^3\lambda} = \frac{lv^2}{C^2\lambda} \qquad ...(7)$$

$l = 1 \times 10^3 \text{cm}$, $\lambda = 5.0 \times 10^{-5} \text{cm}$

$v = 3 \times 10^6 \text{ cm/sec}$

$C = 3 \times 10^{10} \text{cm/sec.}$

$$n = \frac{2lv^2}{\lambda C^2}$$

$$n = \frac{2 \times 1 \times 10^3 \times (3 \times 10^6)^2}{5 \times 10^{-5} \times (3 \times 10^{10})^2}$$

$= 0.4$ fringe.